

Saturn's magnetic field and dynamo

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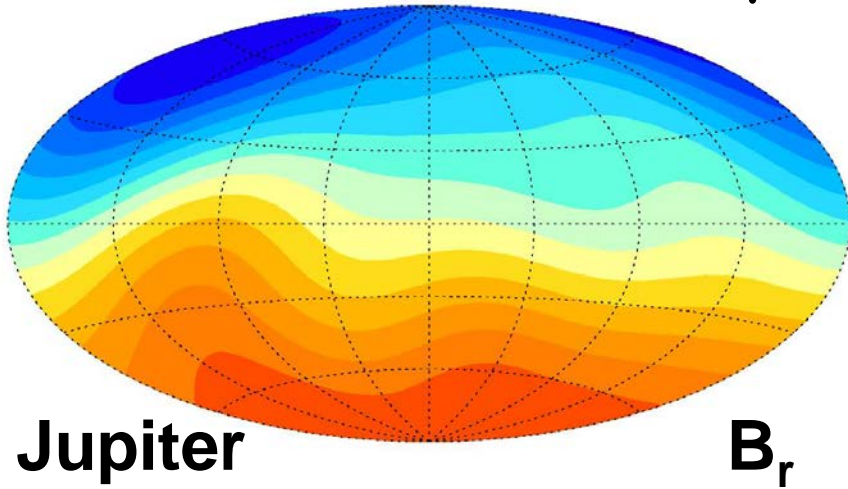
UCLA, Los Angeles, USA

Michele Dougherty

Imperial College, London, UK

Unlike siblings

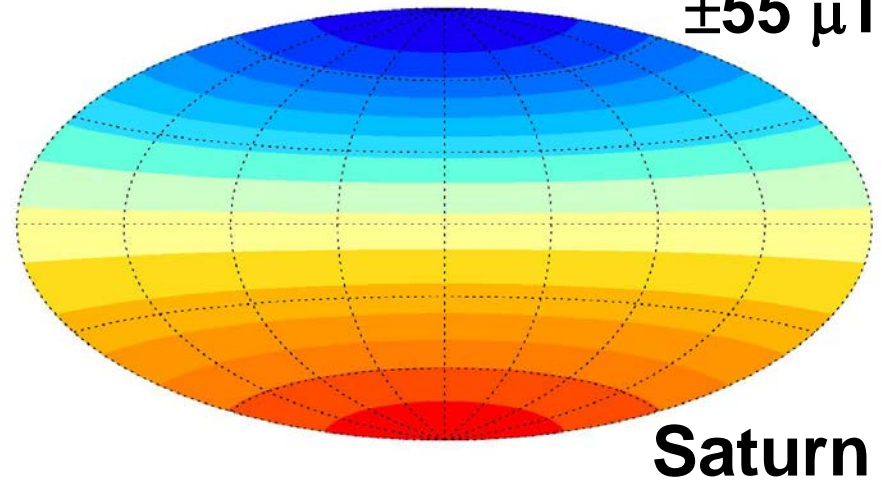
$\pm 1,100 \mu\text{T}$



Jupiter's field 15xEarth's
Similar proportion of
nonzonal to zonal field as
Earth. Dipole tilt 10°

Saturn's field 0.7xEarth's
No nonzonal field
detected so far.
Dipole tilt 0° within error

$\pm 55 \mu\text{T}$



Plan of the talk (and chapter)

1. Observations of Saturn's magnetic field

1.1 Flyby magnetometer measurements: Pioneer 11, Voyager 1 & 2

1.2 Cassini magnetometer measurements

2. Saturn magnetic field models

2.1 External magnetic field and field separation

2.2 Internal magnetic field models

2.3 Search for non-axisymmetric field components

2.4 Secular variation

3. Saturn's dynamo

3.1 Fundamentals of planetary dynamos

3.2 Saturn's internal structure: properties relevant for the dynamo

3.3 Dynamo models with stably stratified layer

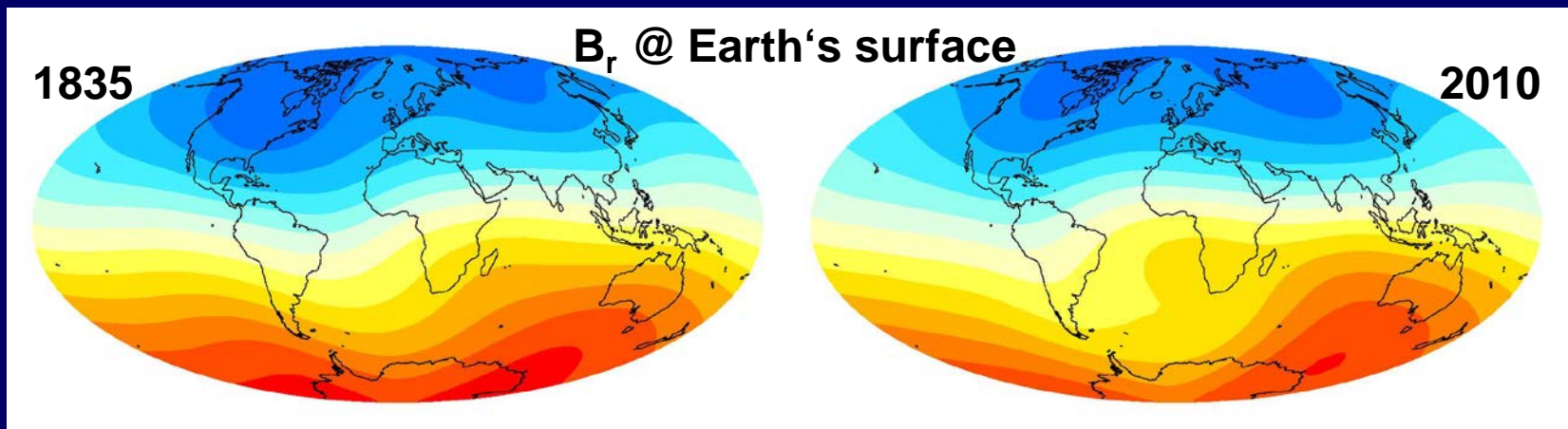
3.4 Other dynamo models and perspectives

Gauss coefficients

$$\mathbf{B} = -grad V$$

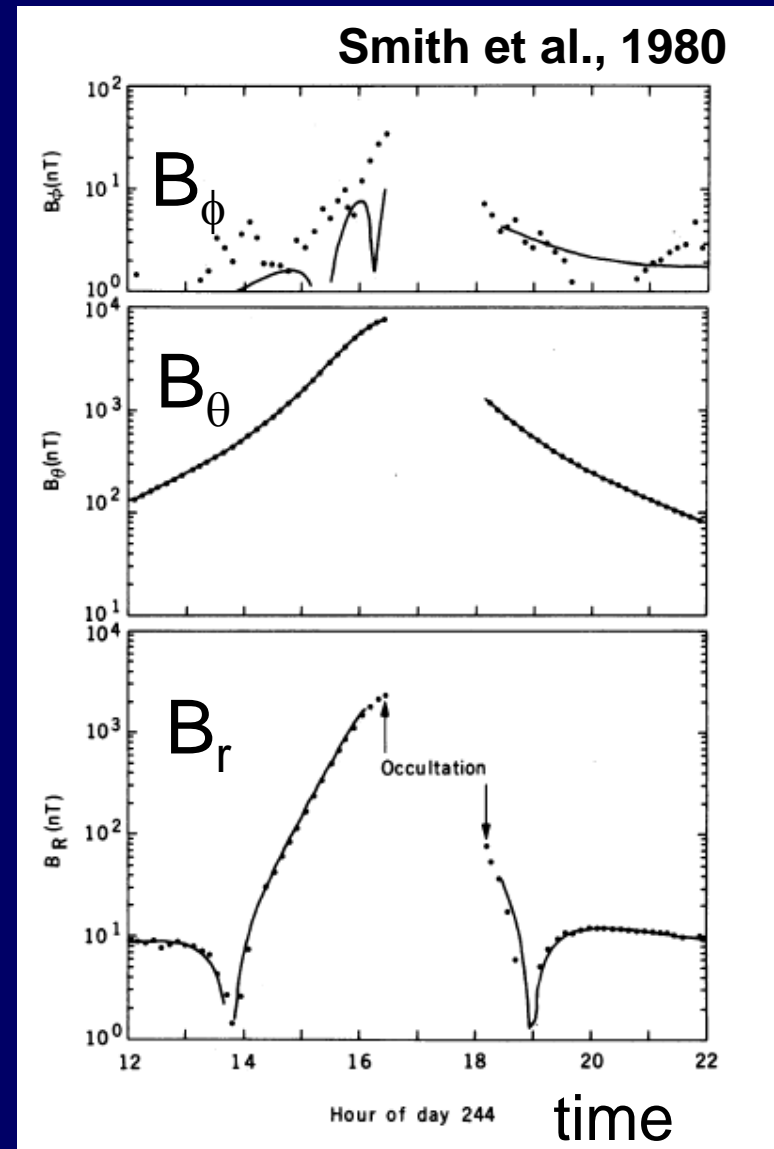
$$V = R_p \sum_{n=1}^{\infty} \left(\frac{R_p}{r} \right)^{n+1} \sum_{m=0}^n P_n^m(\cos \theta) \left(g_n^m \cos m\phi + h_n^m \sin m\phi \right)$$

Representation of field \mathbf{B} by scalar potential V expanded in spherical harmonic functions Gauss, 1839



Saturn flyby measurements

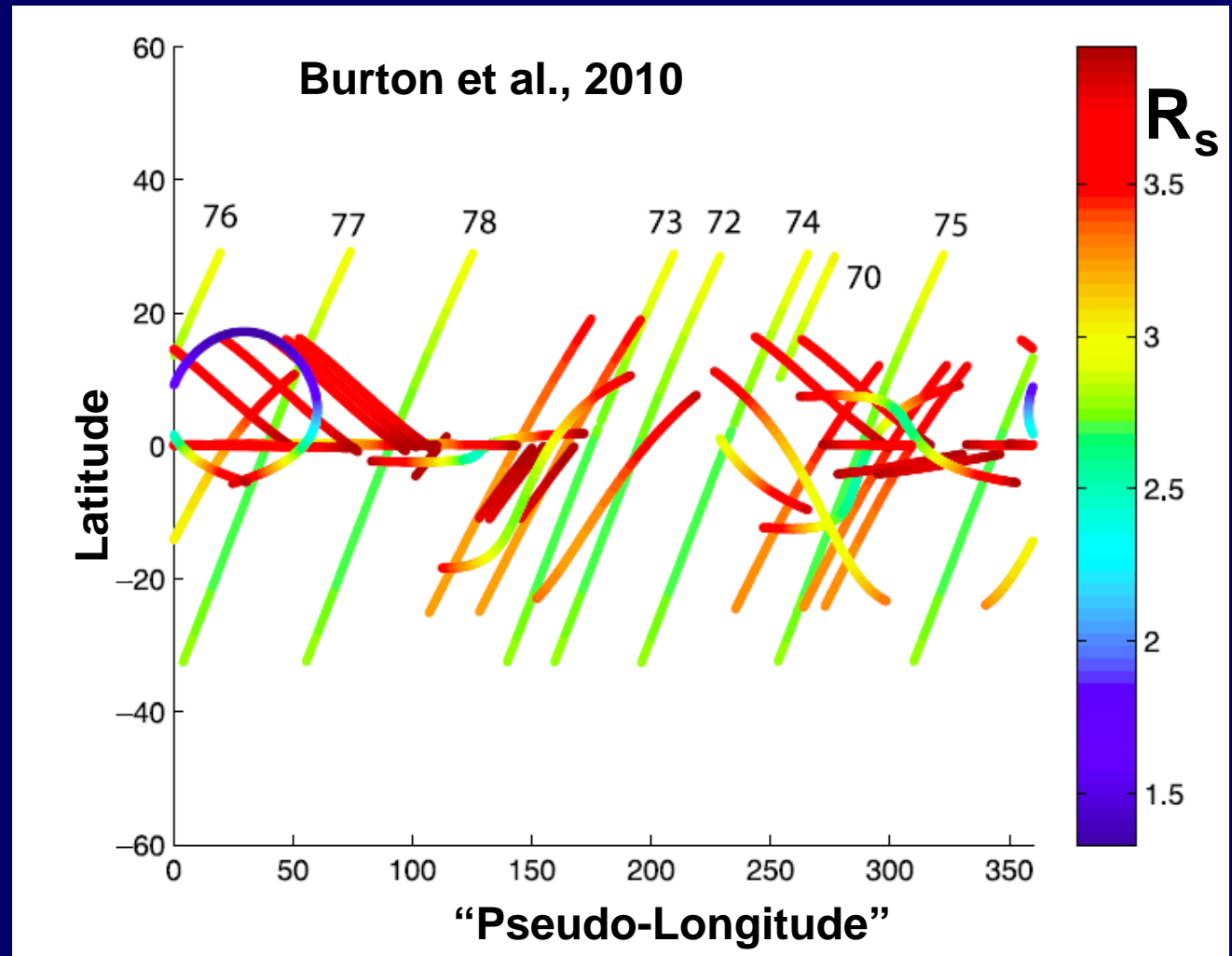
- Pioneer 11 passed at $1.4 R_s$ on 9/1/1979
- He vector magnetometer
- Field weaker and magnetosphere smaller than anticipated
- Dipolar field
- B_ϕ component tiny dipole tilt $< 1^\circ$
- Voyager 1 $3.4 R_s$ 1980
- Voyager 2 $2.7 R_s$ 1981



Cassini SOL & primary mission

$T_{\text{rot}}=10:39:22$

Orbital arcs
inside Ence-
lados L-shell
(3.95 R_s)



External field contribution

Time-variable (stochastic) field: $\sim 5 \text{ nT}$

\Rightarrow Noise

Steady ring current / magnetopause current field: $\sim 15 \text{ nT}$

\Rightarrow To be modelled

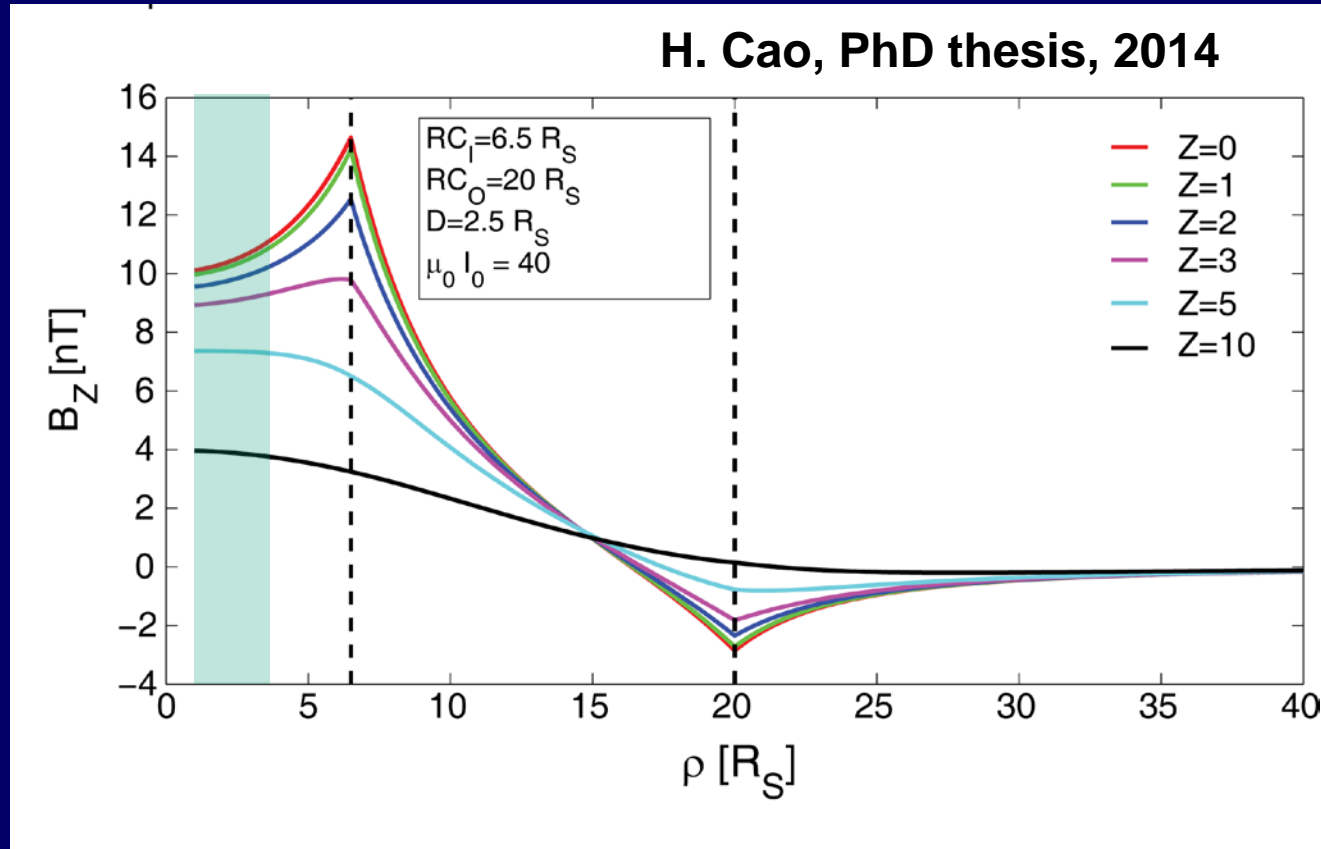
Compare with intrinsic field at $\sim 3 R_s$:

dipole: 800 nT

quadrupole: 20 nT

octupole: 9 nT

Model of ring current field



Inside 3.5 R_S homogeneous within 1 nT

Little local time dependence

Magnetic field model

Table 1. Spherical Harmonic Coefficients for the Axisymmetric Model Derived in This Study^a (in nT)

	This study	Cassini	Cassini(SOI)	Z3	SPV	
g_{10}	21136 (.60)	21153	21162	21248	21225	Dipole
g_{20}	1526 (.37)	1576	1514	1613	1566	Quadrupole
g_{30}	2219 (.90)	2267	2283	2683	2332	Octupole
G_{10}	-11.6	External field of magnetospheric currents (Burton et al., 2010)				
G_{11}	-.04					
H_{11}	-.14					

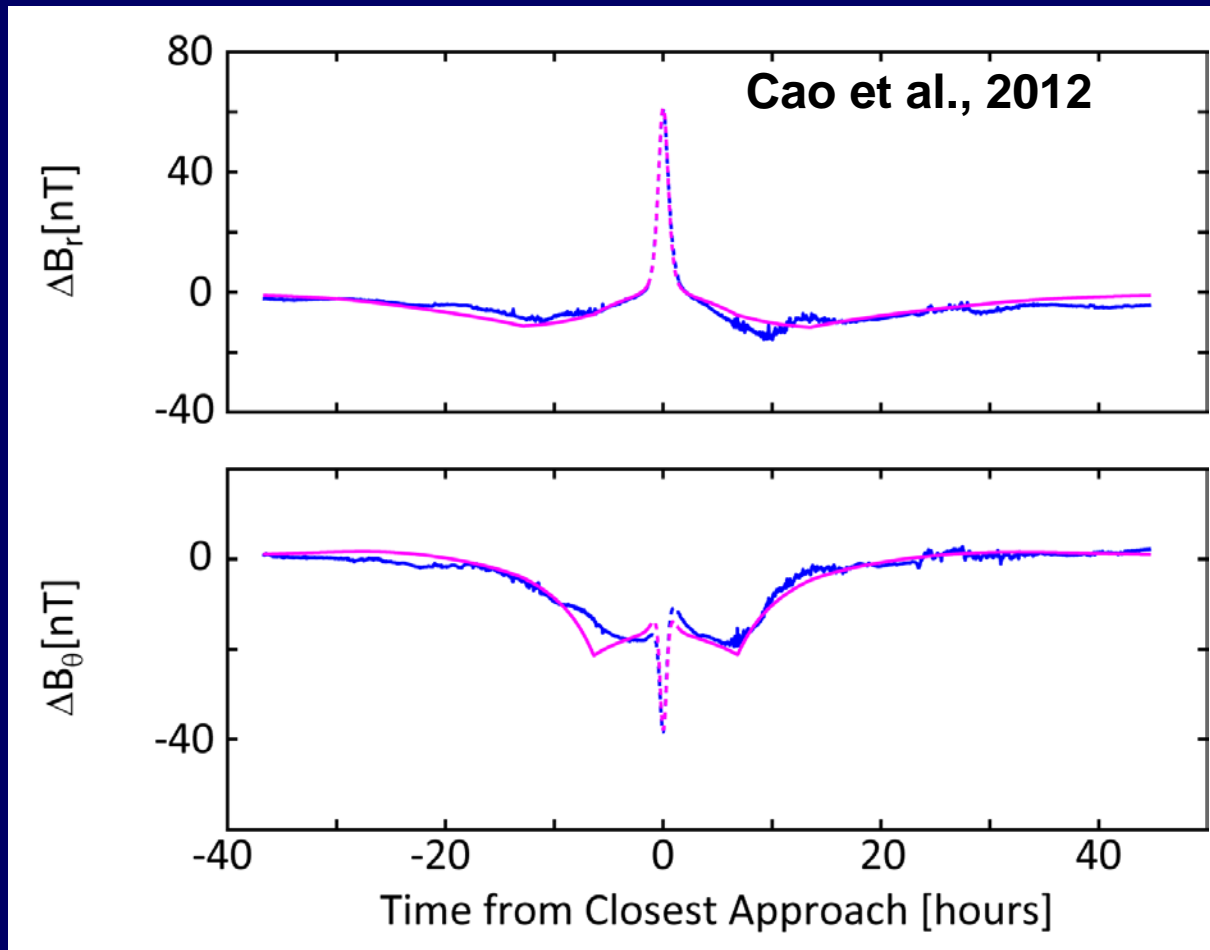
Z3 (Connerney et al., 1984) and SPV (Davis & Smith, 1990) from Pioneer-Voyager data

“Cassini” (Burton et al., 2009) based on 3 yr Cassini data

“This study” (Burton et al., 2009) based on 4 yr Cassini data including orbits 70-78 within 2.7 Rs and latitude coverage up to $\pm 30^\circ$

Later model (Cao et al., 2011) with additional data similar

Beyond the octupole



Blue: Difference between degree 3 internal model and observation at SOI

Broken part of lines: $r < 2.2 R_s$ closest at $1.35 R_s$

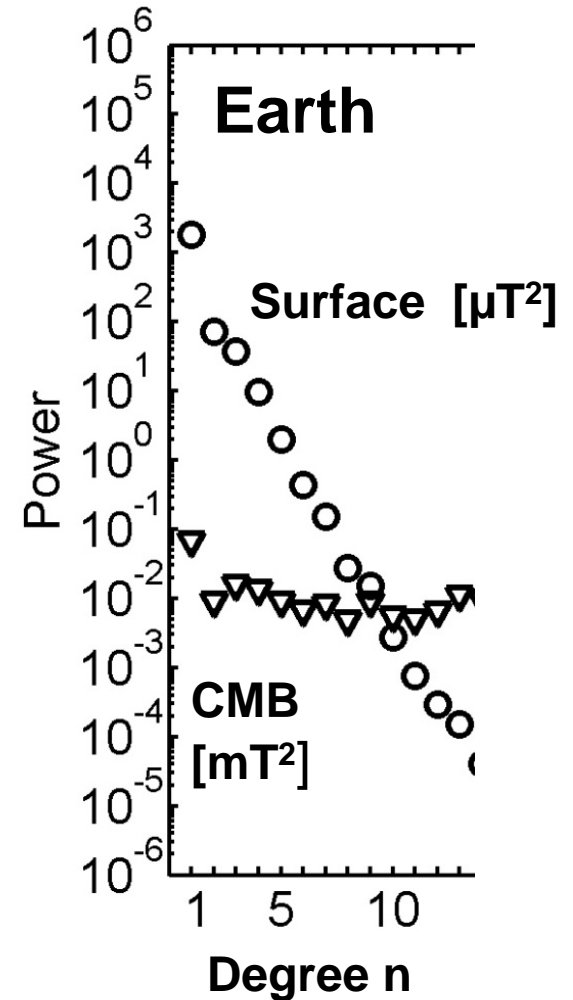
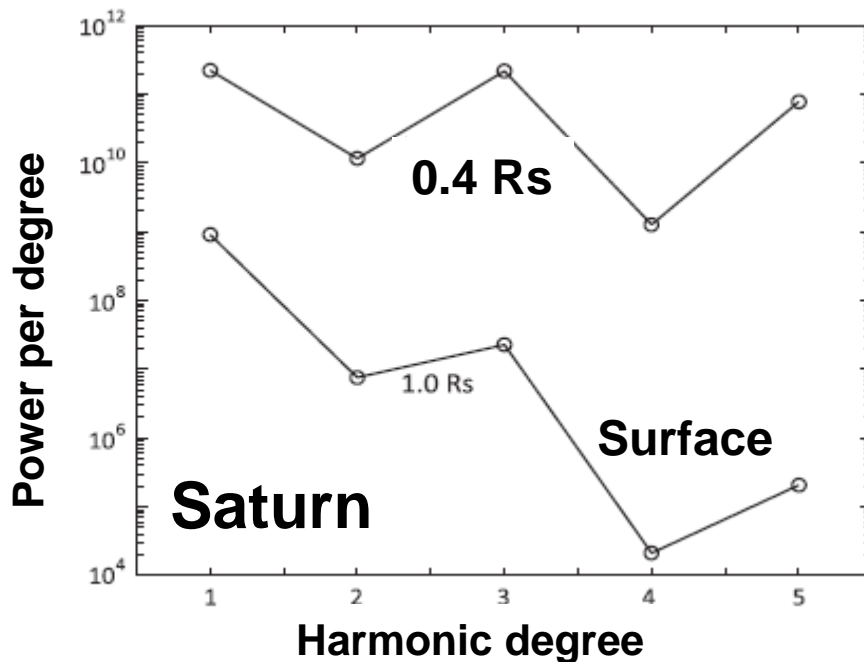
Pink: Fit with model for ring current plus g_{40} and g_{50}

Assumption: Non-zonal terms negligible also at $n=4$ and 5

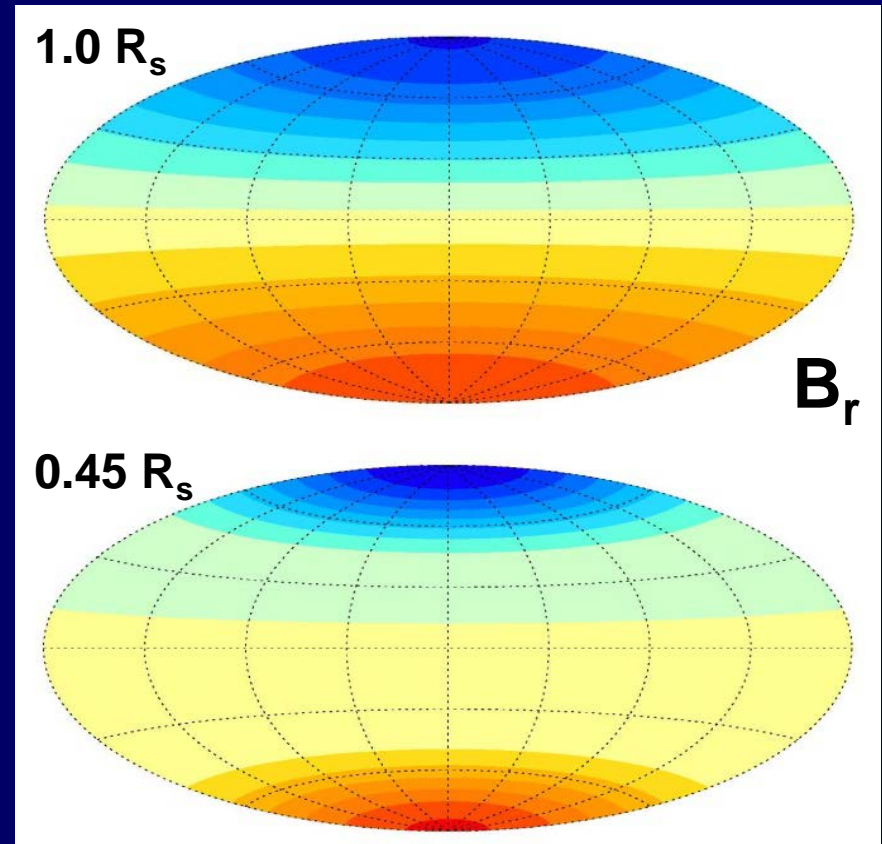
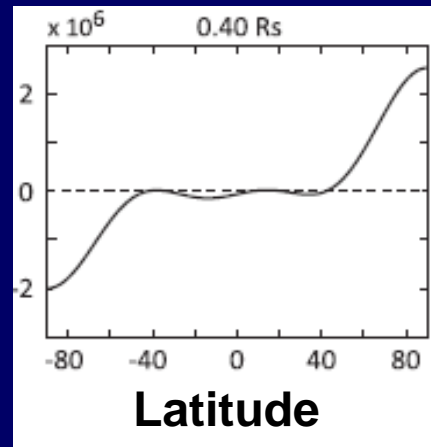
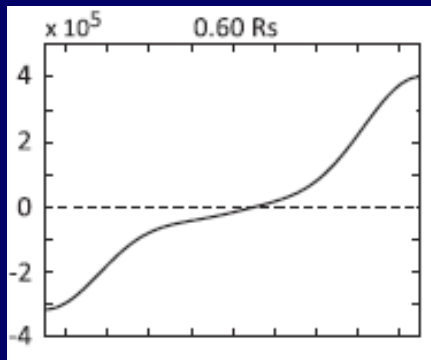
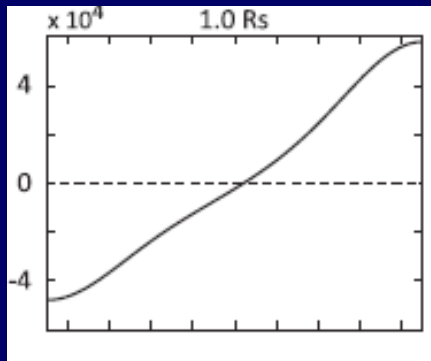
Field up to n=5

	Cassini 3	Cassini 5
g10	21,191 ± 24	21,191 ± 24
g20	1586 ± 7	1586 ± 7
g30	2374 ± 47	2374 ± 47
g40		65 ± 70
g50		185 ± 100

Cao et al., 2012

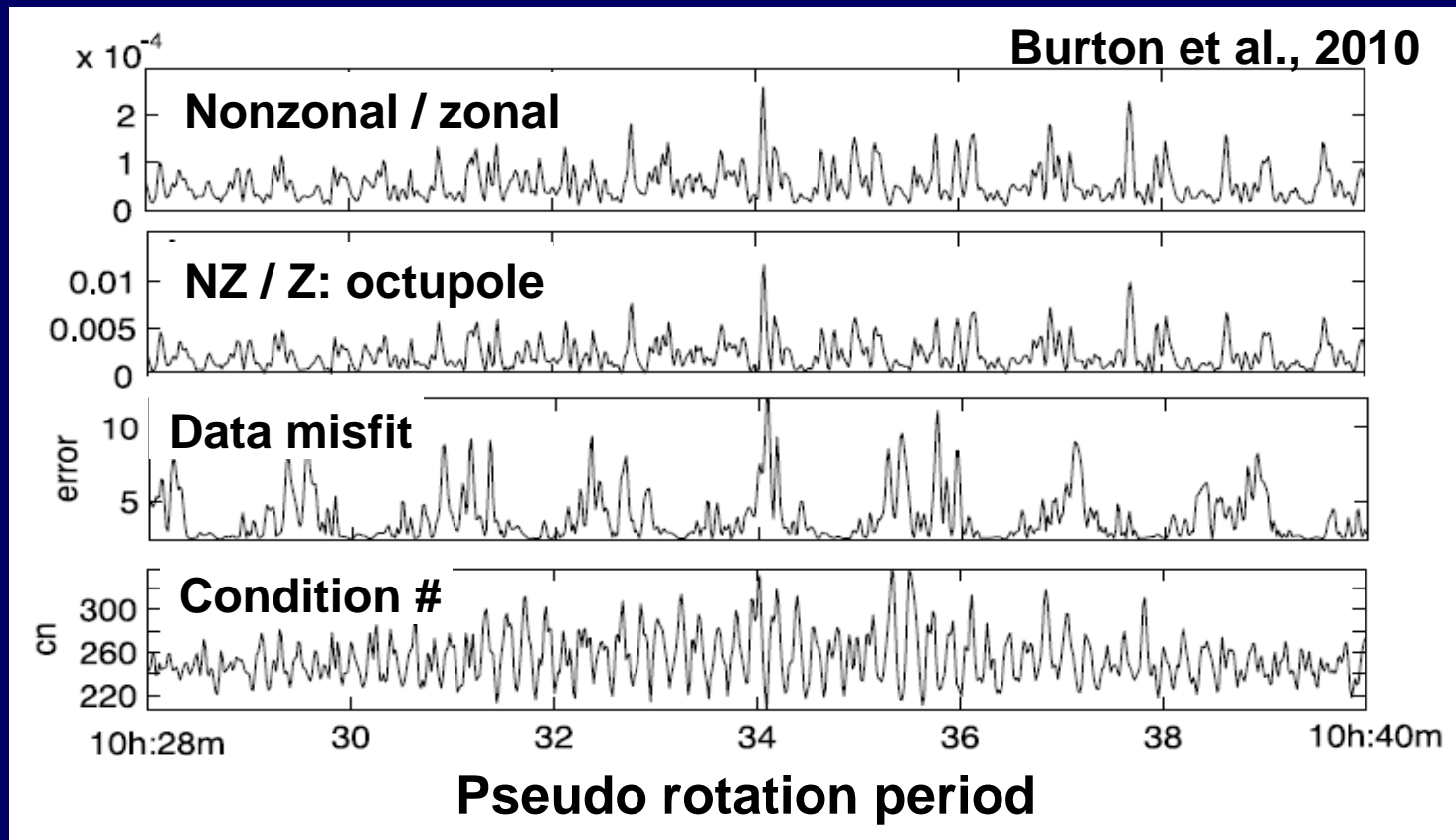


Magnetic flux concentration



Magnetic flux concentrated into polar cap region inside Saturn

Search for non-zonal field



No unambiguous evidence for non-zonal field

(same result by Sterenborg & Bloxham, 2010, and by Cao et al., 2011)

Max. dipole tilt compatible with data is 0.06°

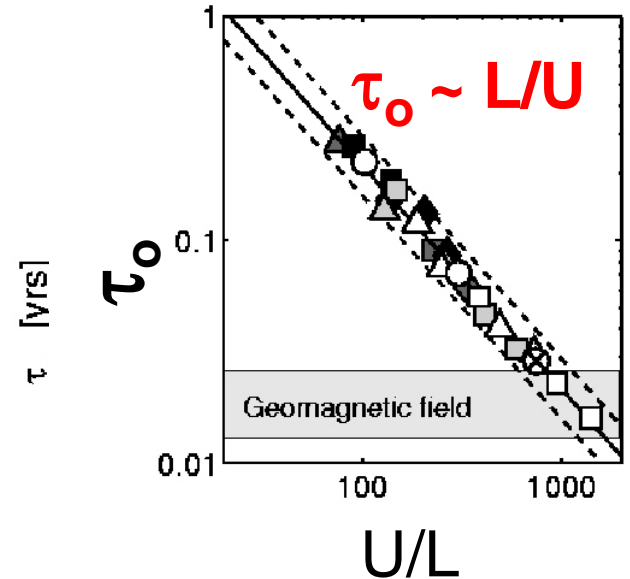
Secular variation: Expectation

Secular variation
time

$$\tau_n = \left(\frac{\sum_{m=0}^n (g_{nm}^2 + h_{nm}^2)}{\sum_{m=0}^n (\dot{g}_{nm}^2 + \dot{h}_{nm}^2)} \right)^{1/2}$$

$$\tau_n = \tau_o / n$$

↑ Dipole off scale



Earth: $U \sim 1$ mm/s $L \sim 3000$ km $L/U \sim 100$ yr $\tau_o \sim 450$ yr

Saturn: $U \sim 10$ mm/s $L \sim 30,000$ km $\Rightarrow \tau_o \sim 450$ yr

\Rightarrow Similar rates of secular variation for Earth and Saturn ?

Secular change Pioneer - Cassini

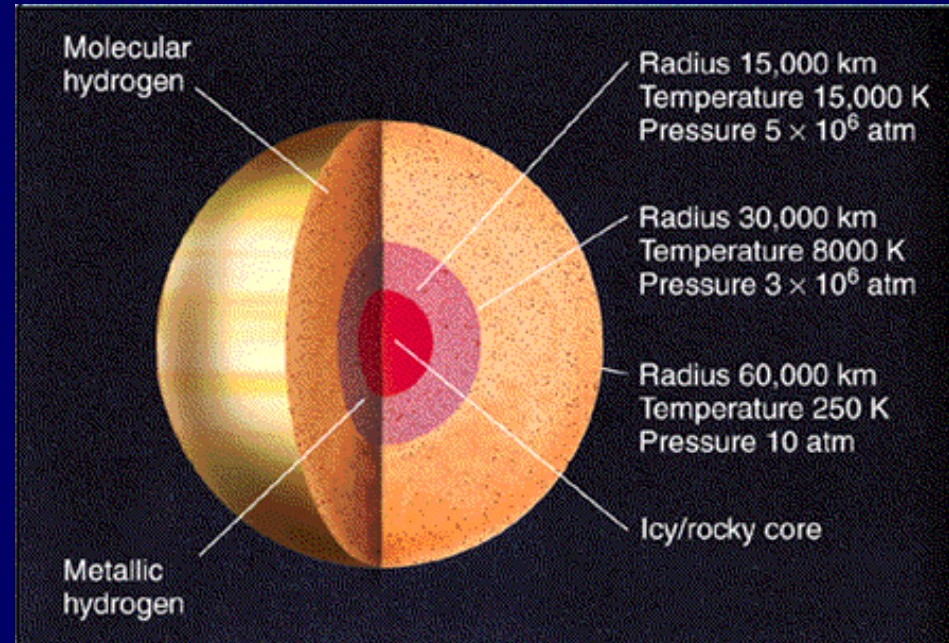
Coefficients	Cassini (Rev 3-126)	SPV
g_1^0	$21,191 \pm 24$	21,225
g_2^0	1586 ± 7	1566
g_3^0	2374 ± 47	2332

	g_{10}	g_{20}	g_{30}	
Earth	19.6	16.7	4.2	nT/yr
Saturn	-1.2 ± 1.6	0.7 ± 0.5	1.5 ± 3.2	nT/yr

Secular variation small, or zero within error

Requirements for dynamo

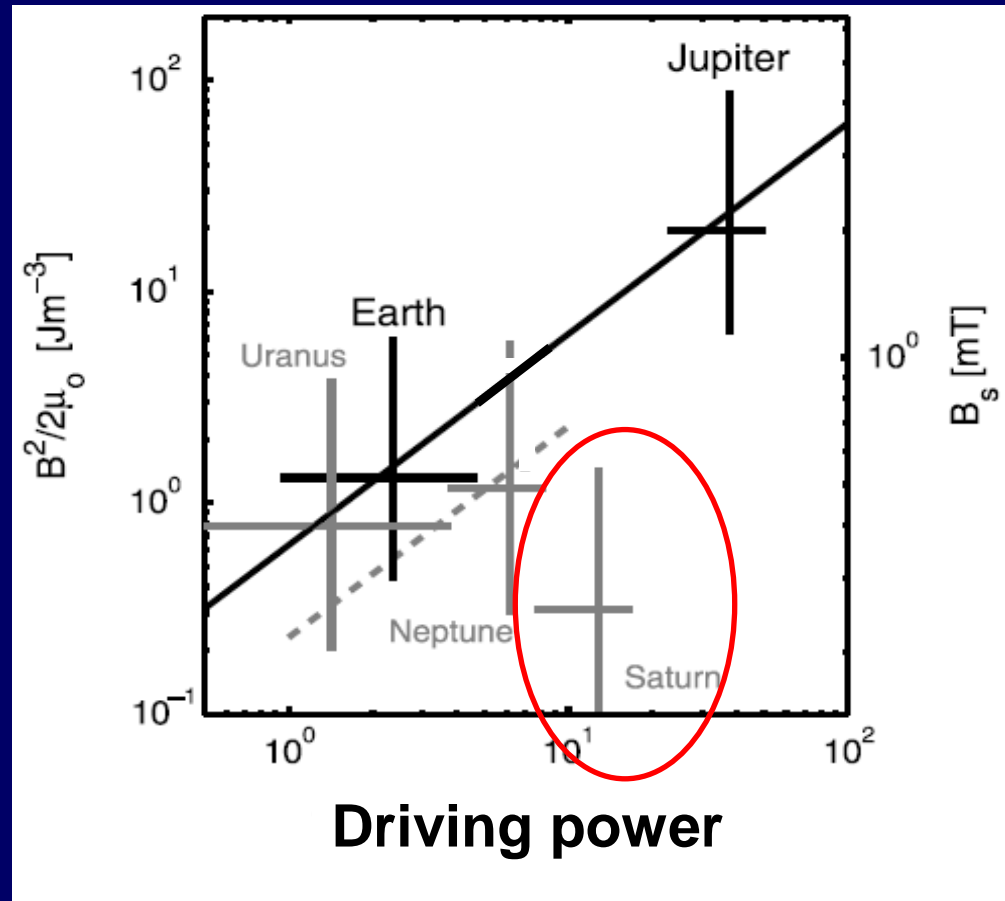
- **Fluid electrically conducting layer**
(metallic hydrogen)
- **Sufficiently rapid motion:**
magnetic Reynolds no.
 $Rm = UL/\eta > 50$
(thermal convection,
 $Rm \sim 10^4 - 10^5$)
- **Suitable pattern of motion, e.g. helical**
(Coriolis forces: Saturn is a rapid rotator)



Excess heat flow

Earth	0.08 W/m ²
Jupiter	4.5 W/m ²
Saturn	2.0 W/m ²

Scaling field strength



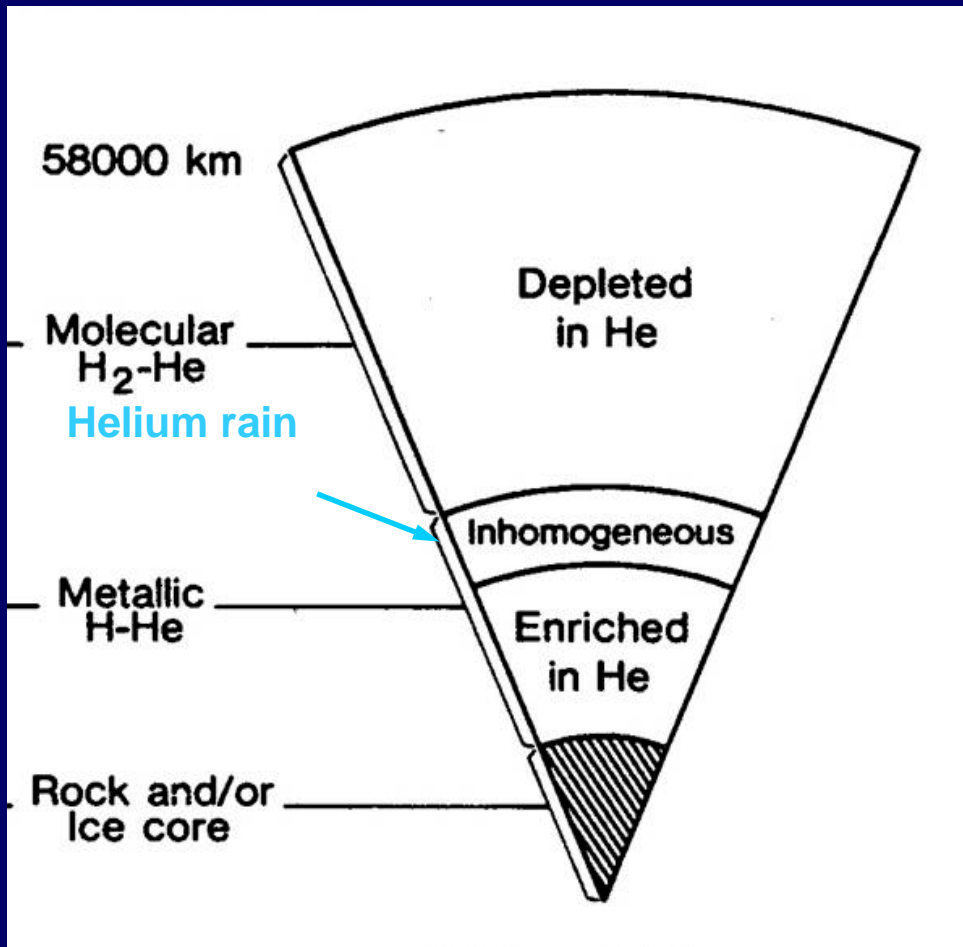
Field strength \propto cubic root of available energy flux

Christensen & Aubert, 2006; Aubert et al., 2009; Christensen et al., 2009; Christensen, 2010

What must a good Saturn dynamo model be able to explain ?

- The field strength or dipole moment of Saturn (lower than expected from scaling laws)
- The extreme degree of axisymmetry (nonwithstanding Cowling's theorem)
- The concentration of magnetic flux into the polar regions (at depth inside the planet)
- The small rate of secular variation

Stevenson's model

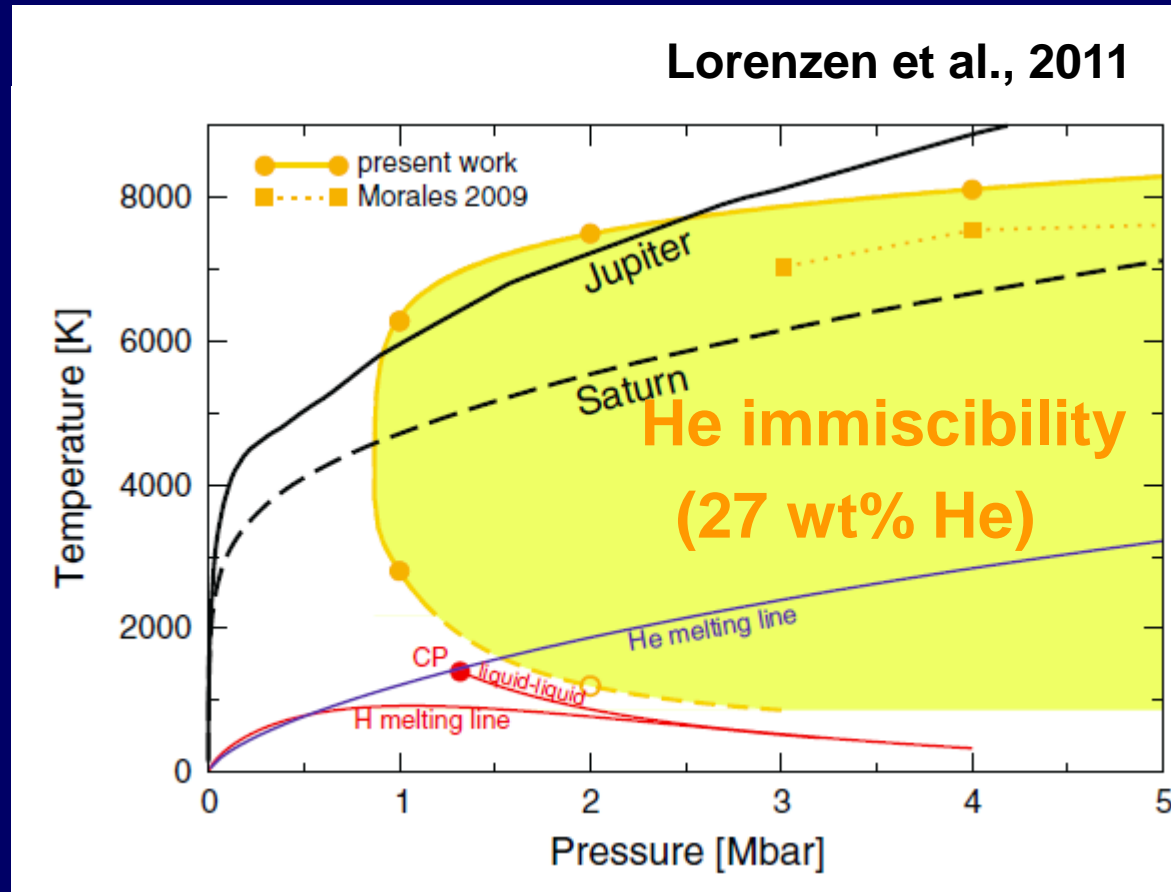


He rain depletes upper layer in helium and provides extra energy to enhance luminosity (or retard cooling)

He rain creates electrically conducting stably stratified layer above the dynamo

(Stevenson, 1980)

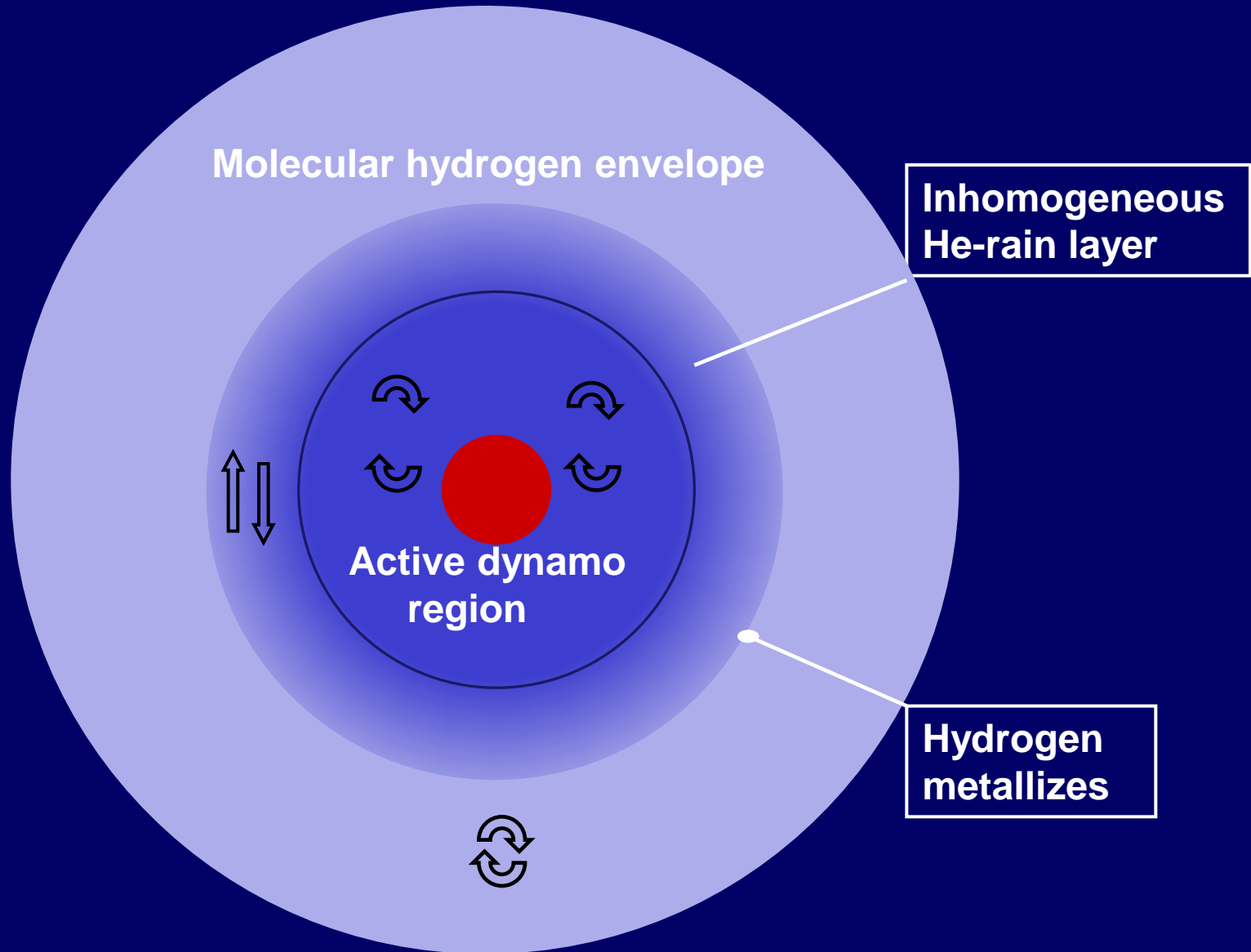
He immiscibility



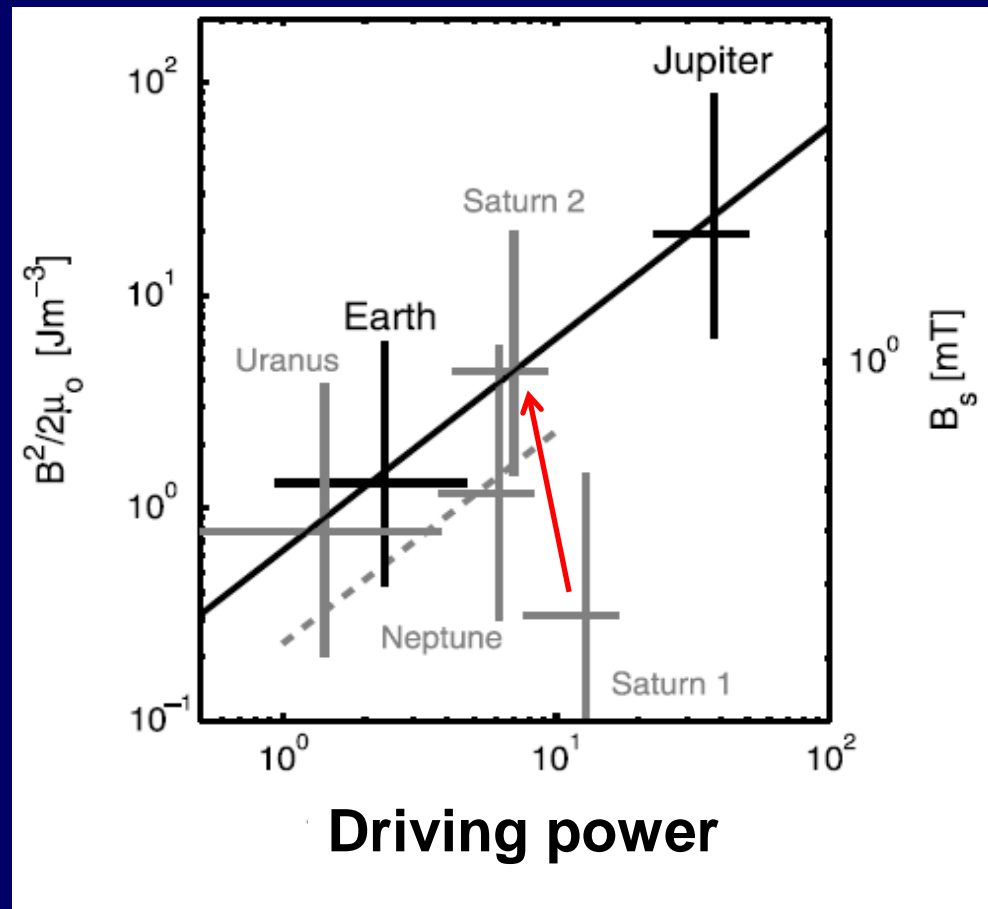
Pseudo rotation period

Ab-initio calculations suggest that He may rain out all the way to the rocky core (and form He ocean above core)

Dynamo below He-rain layer



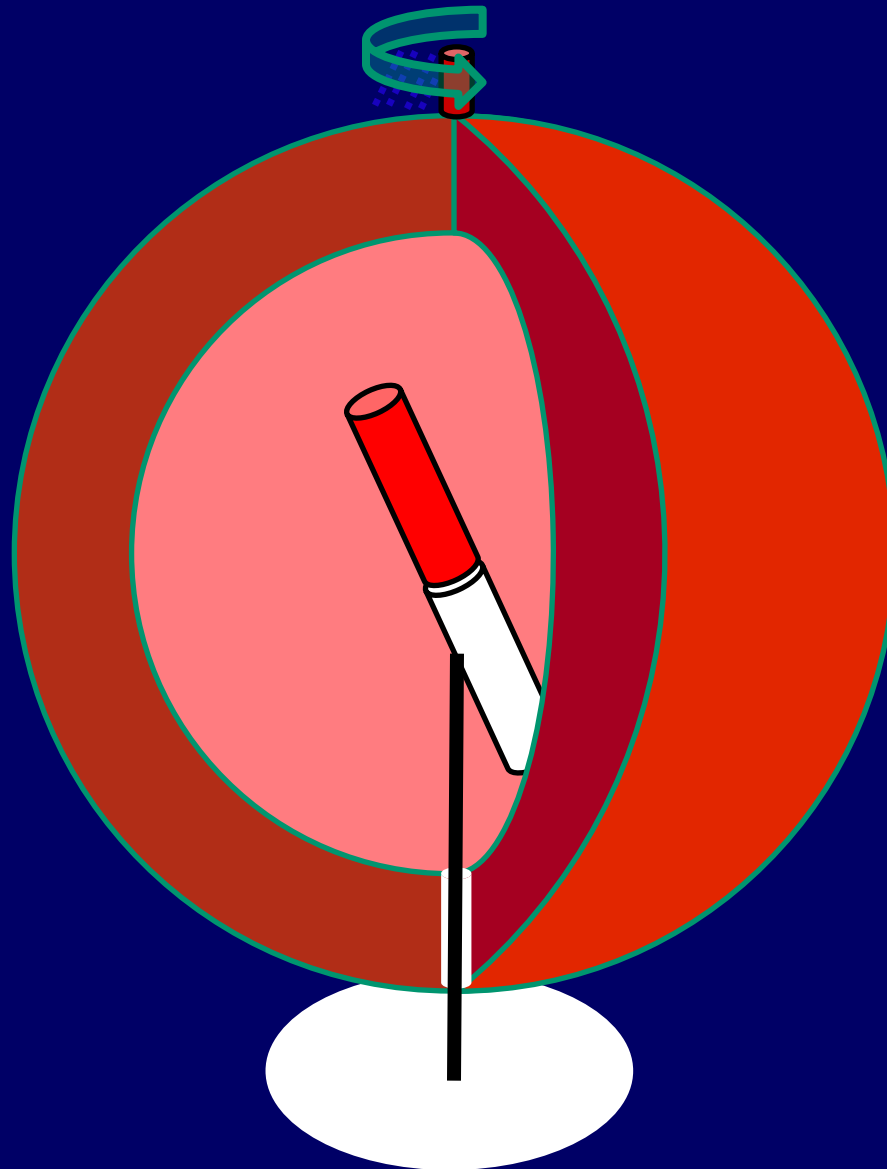
Field strength for deep dynamo



With the top of Saturn's dynamo at 0.4 R_s rather than at 0.6 R_s the observed field complies with the scaling relation

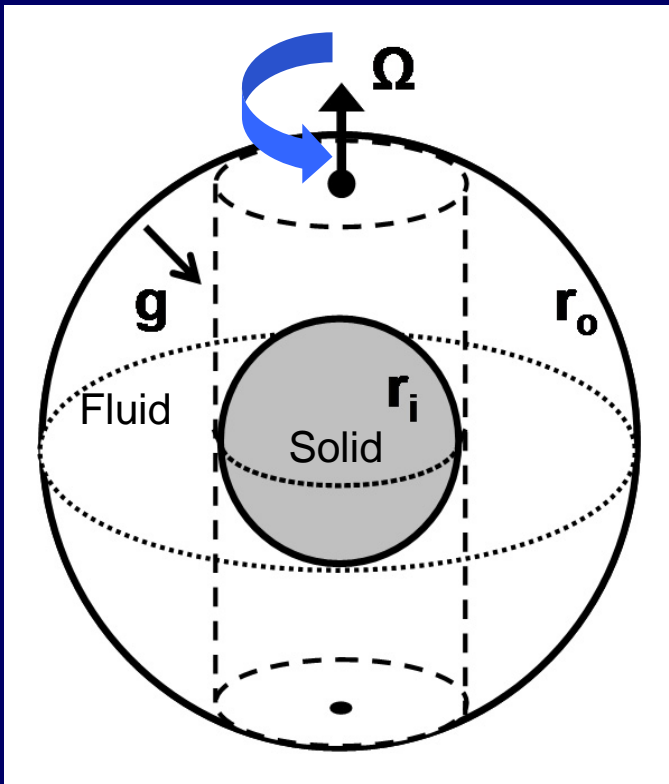
Christensen, 2010

A Gedankenexperiment



Numerical dynamo models

- Convection-driven MHD dynamo models in rotating and electrically conducting spherical shells
- Demonstration of principle: Influence of conducting stably-stratified layer above dynamo region



$$\rho \left(\frac{\partial \vec{u}}{\partial t} + \vec{u} \cdot \nabla \vec{u} \right) = -\nabla p + \rho \vec{g} + \nabla \cdot \vec{\tau} + \vec{J} \times \vec{B}$$

Inertia
Coriolis
Viscosity
Buoyancy
Lorentz

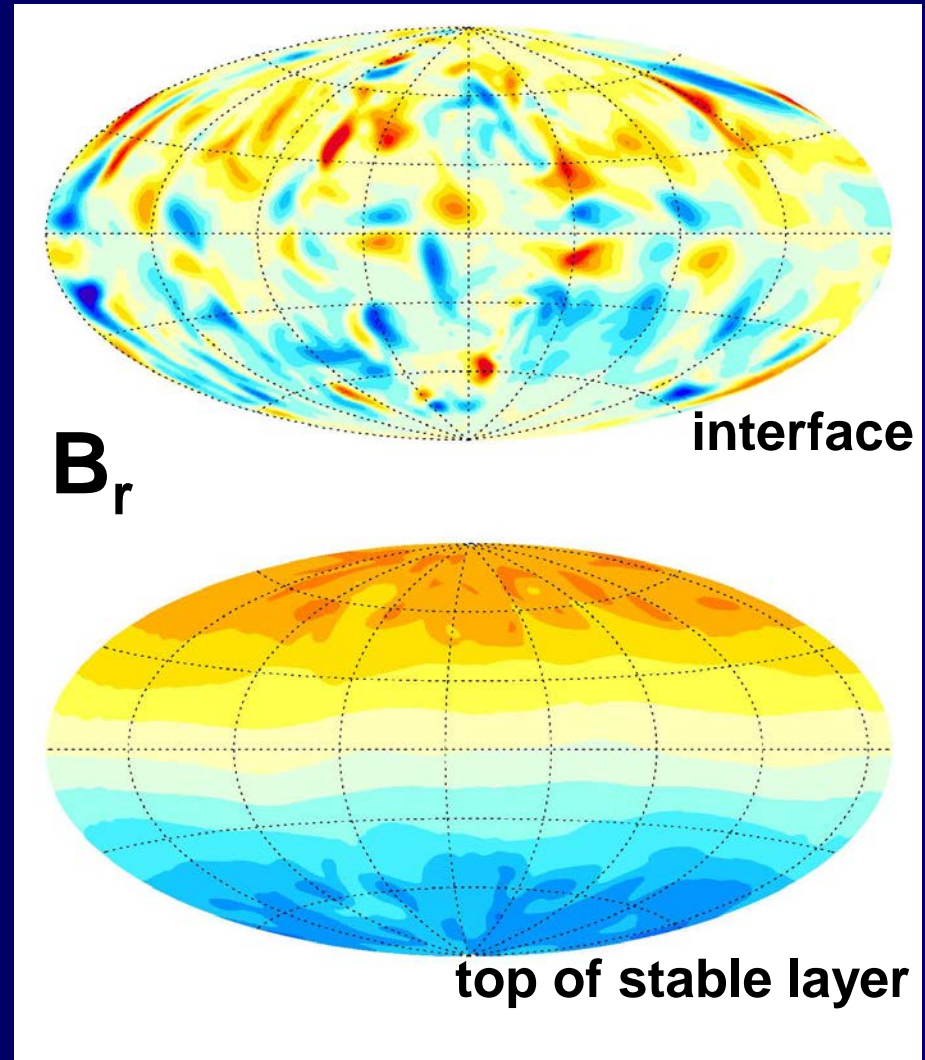
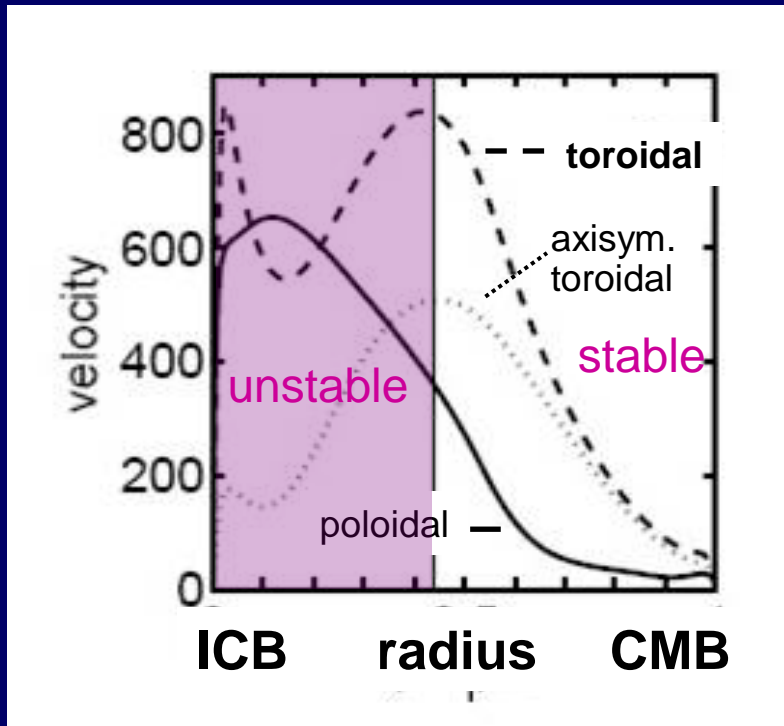
$$\rho c_p \left(\frac{\partial T}{\partial t} + \vec{u} \cdot \nabla T \right) = \nabla \cdot \vec{q} + \vec{J} \cdot \vec{E}$$

Advection
Induction
Diffusion

$$\frac{\partial \vec{B}}{\partial t} + \vec{u} \cdot \nabla \vec{B} = \nabla \cdot \vec{\eta}$$

Magnetic field snapshots

Dynamo generates strong nonzonal field component, which is largely filtered out by stable layer

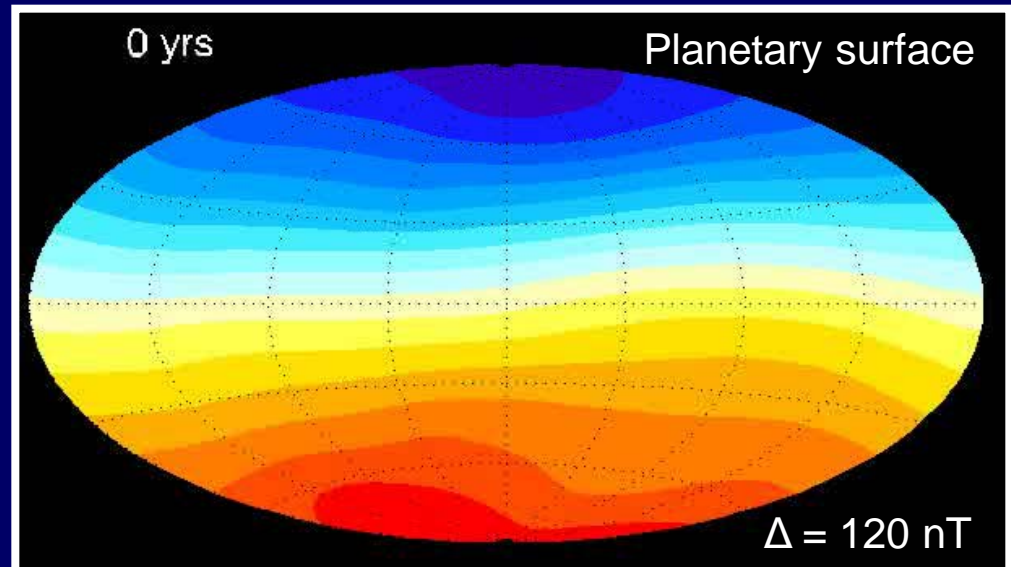
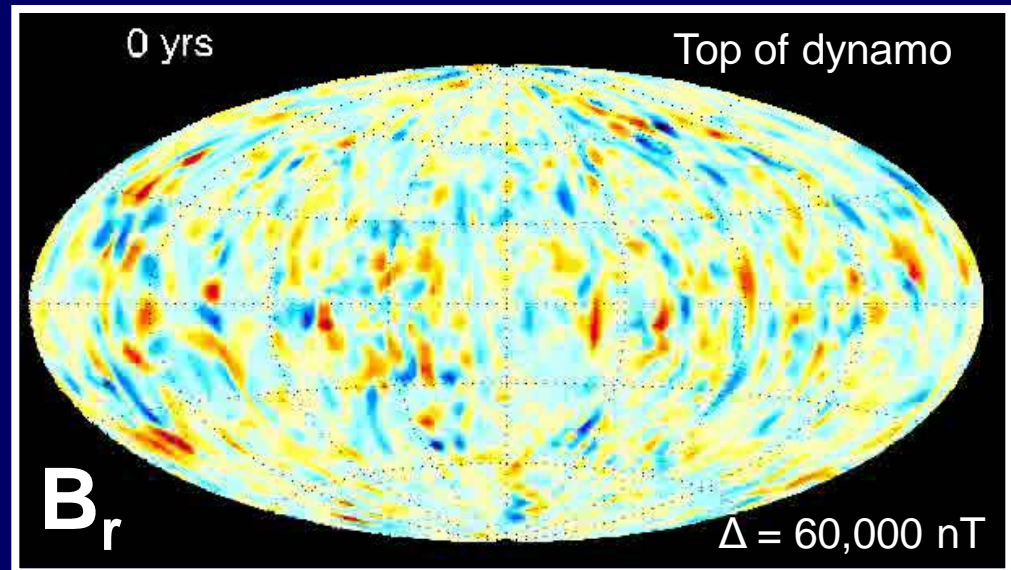


Mercury model with stable layer

- Field inside active dynamo region is strong, small-scaled and varies rapidly.
- Field at planetary surface is weak, large-scaled, and varies slowly. Field strength agrees with observation.

Christensen, 2006

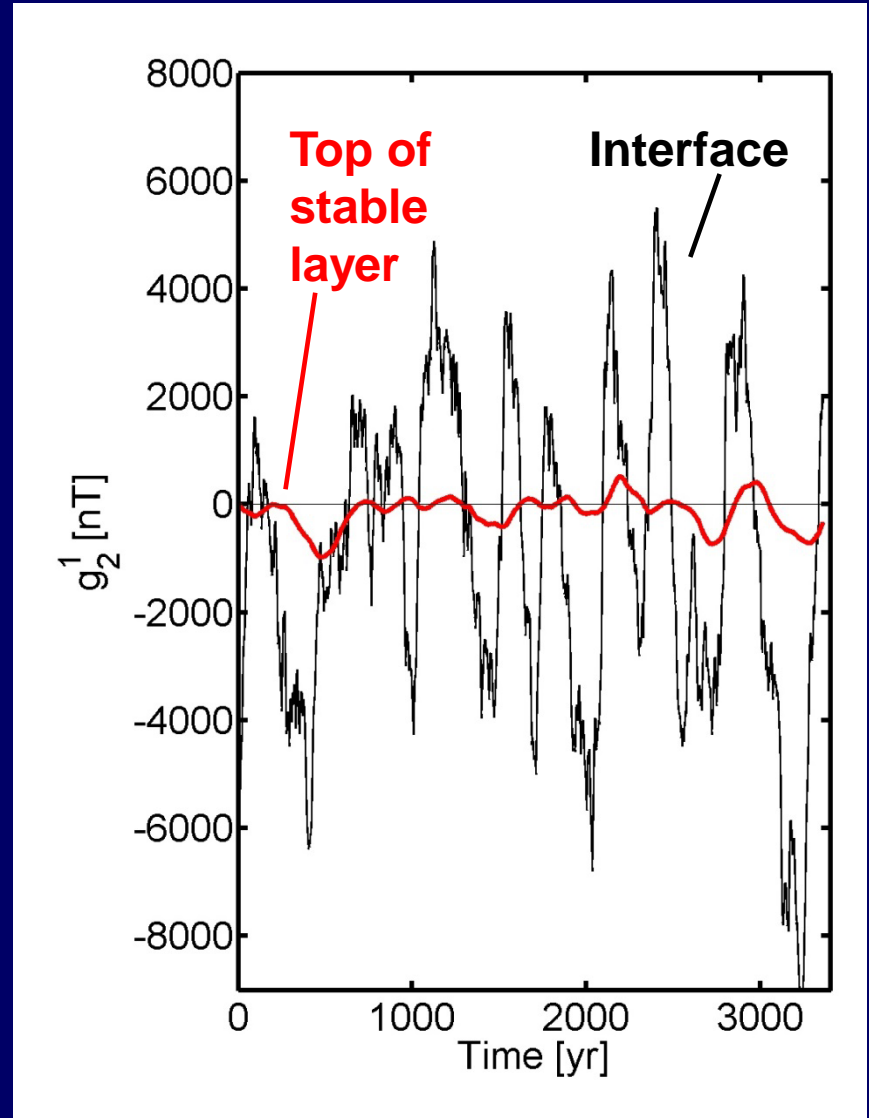
Christensen & Wicht, 2008



Skin effect

- Time series Gauss coefficient g_{21}
- Elimination of high frequencies
- Damping of low frequencies
- Phase shift

(Ganymede model,
Christensen, in prep.)



Score of model with dynamo below stable layer

- **Field strength**



- **High degree of axisymmetry**



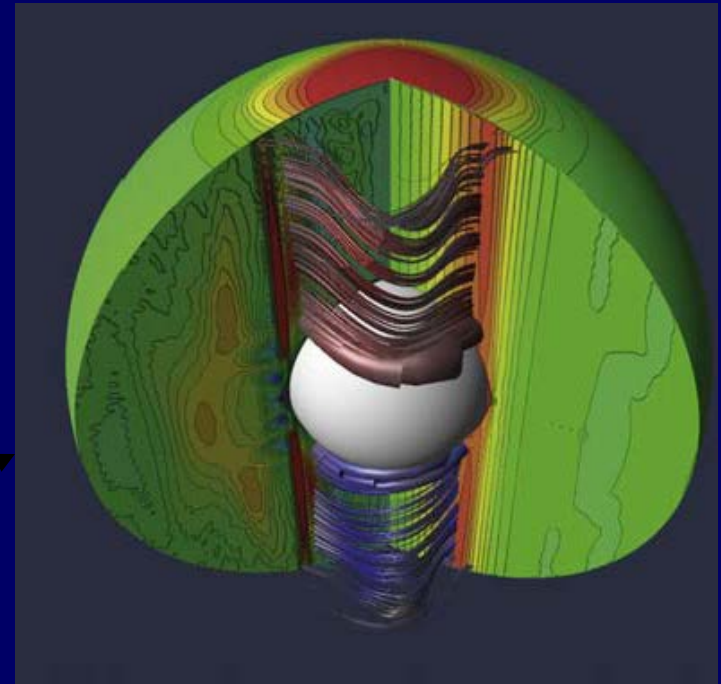
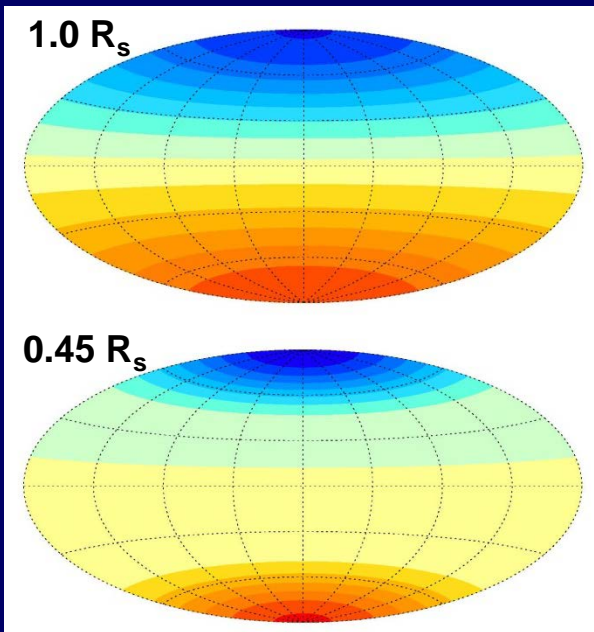
- **Small rate of secular variation**



- **Polar flux concentration**



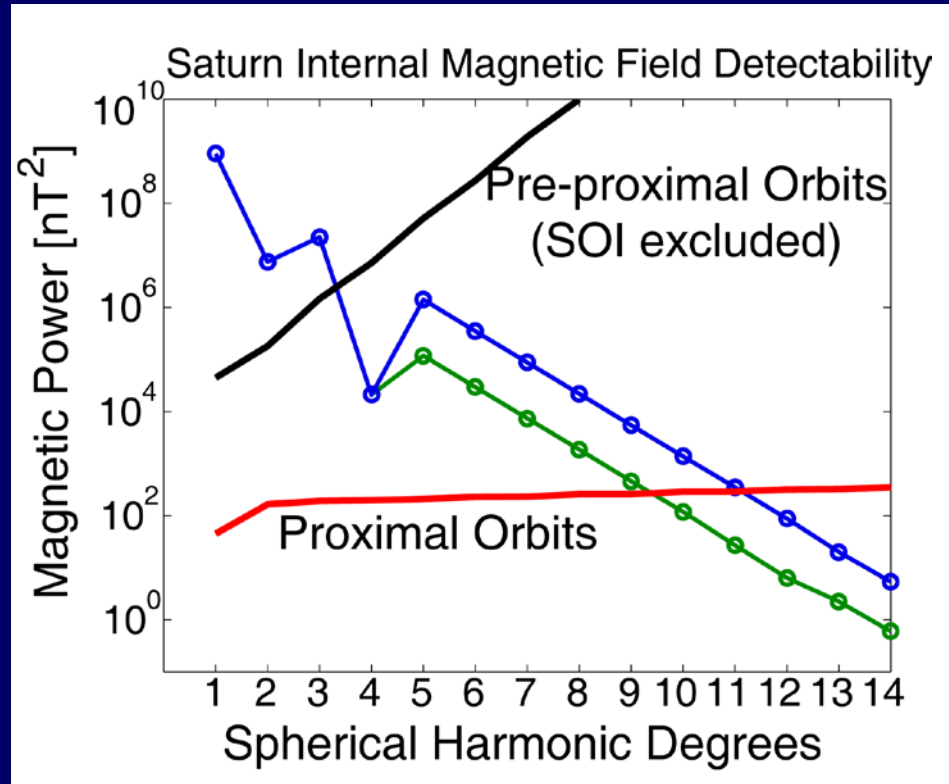
Polar flux concentration: Taylor-Couette dynamo



Taylor-Couette dynamo: flow driven by faster rotation of inner core: Strong flux emanating at poles (Cao et al., 2012)

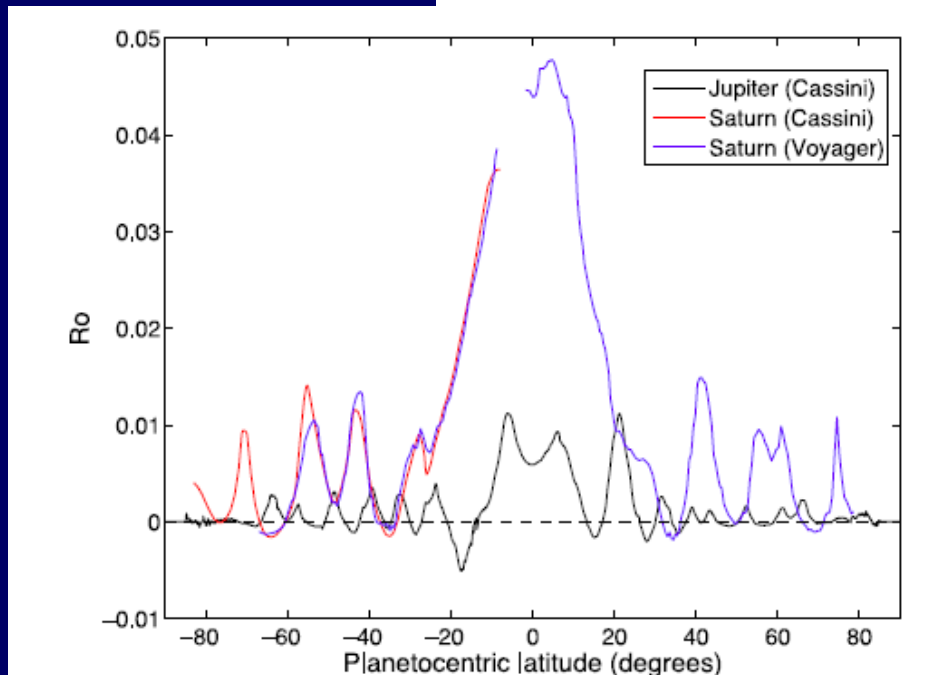
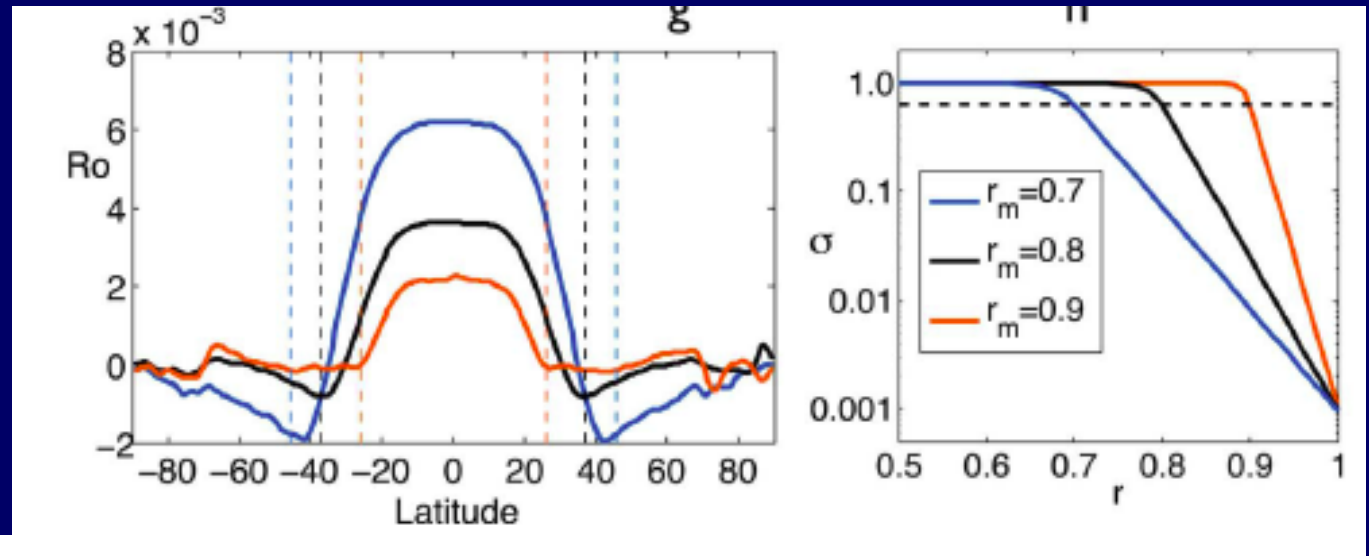
But unclear what would drive differential rotation in Saturn

Outlook: Proximal mission



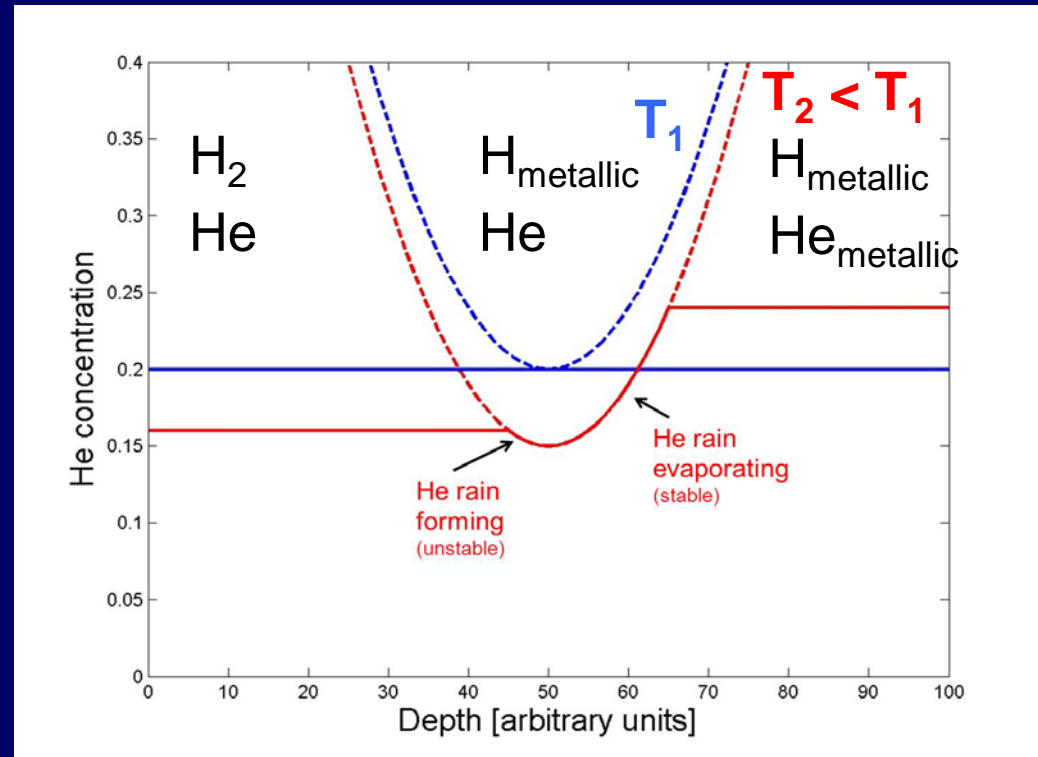
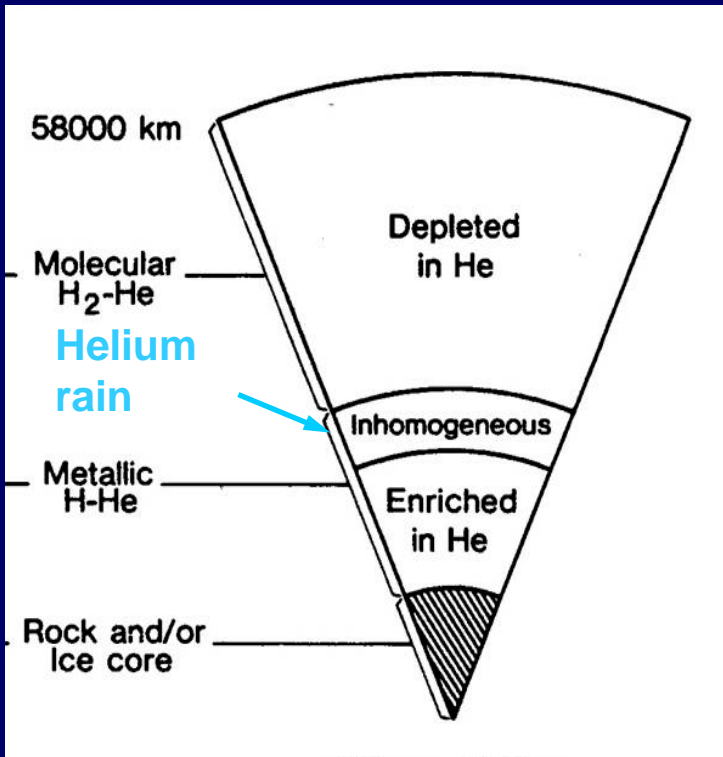
- If power continues to drop off as odd degrees 1, 3, 5 then the field up to degree 9 at least can be determined
- Nonzonal field components with $g_{nm} > 15 \text{ nT}$ can be detected

Zonal winds and dynamo ?



Gomez-Perez & Heimpel., 2011

Stevenson's model



He rain depletes upper layer in helium and provides extra energy to enhance luminosity (or retard cooling)

He rain creates electrically conducting stably stratified layer above the dynamo

(Stevenson, 1980)